

Angular Size in a Static Universe

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ABSTRACT

In principle the geometry of the universe can be investigated by measuring the angular size of known objects as a function of distance. Thus the distribution of angular sizes provides a critical test of the stable and static model of the universe described by Crawford (1991,1993) that has a simple and explicit relationship between the angular size of an object and its redshift. The result is that the agreement with observations of galactic diameters and the size of double radio sources with the static model is much better than the standard (Big Bang) theory without evolution. However there is still a small discrepancy at large redshifts that could be due to selection effects.

Subject headings: cosmology: quasars: angular size: galaxies

1. Introduction

The testing of cosmological theories by investigating the dependence of angular size as a function of redshift has slowly gained impetus due to the increasing number of objects with large redshifts. Because size is such a simple concept it should be an excellent tool to investigate the geometry of the universe. However there is a paradox in that the observational results appear to fit a static Euclidean model where angular size is inversely proportional to redshift and they cannot be reconciled with the standard cosmological model without invoking evolution or some other effect such as size dependence on magnitude together with selection effects. There is no general agreement (Nilsson et al. 1993) on what the evolution or special effects should be. For a given set of data it is usually possible to find an evolution function that provides a good fit. However the freedom provided by the use of an evolution function means that the use of angular size data to test the standard model reduces to arguments about whether the evolution function is reasonable or whether it can be disentangled from other z dependent effects. Nilsson *et al* argue that because of a power-size anticorrelation their data is consistent with no cosmological evolution. The reality of this power-size anticorrelation depends on the cosmological model and for the standard model on what value of the deceleration parameter (q_0) is chosen.

Here it will be shown that the angular size data is in agreement with the stable static cosmology without requiring any evolutionary function. My point is that not that the standard model with or without evolution is wrong but that the data is also consistent with a static cosmology that has no extra free parameters or functions. The basic equations for the static cosmology are derived and then the extensive double radio source data given by Nilsson *et al.*, (1993) are analysed. Except for selection effects the agreement is excellent. Finally the static model is also compared with galaxy size data from Djorgovski and Spinrad (1981) and radio data from Kapahi (1987).

2. The Cosmological Model

The cosmological model described in Crawford (1991,1993) has the same geometry as the static Einstein universe (and incidentally the same as a closed big bang model that is not expanding) which is that for a three dimensional surface of a four dimensional hyper-sphere. In this theory the Hubble redshift is caused by a gravitational interaction with the inter-galactic plasma (Crawford 1987). The redshift is a function of the distance

and the density of the plasma. Provided the plasma density does not vary significantly the redshift is a good measure of the distance to an object.

Since the cosmological model is static and is not evolving it obeys the *perfect cosmological principle* (Bondi & Gold 1948) in which there is both spatial and time isotropy. This does not prevent objects such as galaxies or quasars from evolving. What it does require is that their creation and evolution is independent of their location in space and in time. Hence any sampling of the universe over a sufficiently large scale should not detect any variation in the average characteristics of the objects such as size, luminosity or density. The application of this principle to quasars, galaxies or radio sources requires that their average linear size should be independent of their redshift. A critical test that would refute the static cosmology is unequivocal evidence of a change in linear size with redshift that is not due to selection effects.

From Crawford (1991) the redshift ($z = \lambda_0/\lambda_e - 1$) for a photon that has travelled a distance r is given by

$$z = \exp\left(\frac{Hr}{c}\right) - 1, \quad (1)$$

where H is the Hubble constant. For a static model H is not the expansion velocity but a measure of the observed redshift per unit distance. One of the major results of the model is to relate the Hubble constant to the universe's radius R . From Crawford (1993) we get $R = \sqrt{2}c/H$ which provides the basic relationship

$$r = \frac{R}{\sqrt{2}} \ln(1 + z). \quad (2)$$

Let a source of radiation have a luminosity $L(\nu)$ (W.Hz⁻¹) at the emission frequency ν . Then if energy is conserved the observed flux density $S(\nu_0)$ (W.m⁻².Hz⁻¹) at distance r is the luminosity divided by the area which is

$$S(\nu_0) = \frac{L(\nu)}{4\pi R^2 \sin^2(r/R)}.$$

However because of the gravitational interaction there is an energy loss such that the received frequency ν_0 is related to the emitted frequency ν_e by equation (1) and is

$$\nu_0 = \nu_e \exp(\sqrt{2}r/R) = \nu_e/(1 + z).$$

The loss in energy means that the observed flux density is decreased by a factor of $1 + z$. But there is an additional bandwidth factor $d\nu_e = (1 + z)d\nu_0$ that tends to balance the energy loss factor. The balance is not perfect because the source is observed at a different part of its spectrum from that for a similar nearby source. The correction for this spectral

offset is called the K-correction and for radio sources the spectrum is often approximated by a power law with the form $L \propto \nu^\alpha$. With this power law spectrum the apparent luminosity including energy loss and bandwidth corrections is

$$S(\nu_0) = \frac{L(\nu_0)(1+z)^\alpha}{4\pi R^2 \sin^2(r/R)}. \quad (3)$$

For the geometry of the hyper-sphere the observed angular size of an object (small angle approximation) with projected linear size D at a distance r is given by

$$\theta = \frac{D}{R \sin(r/R)}, \quad (4)$$

where R is the radius of the universe. Hence the angular size as a function of redshift is

$$\theta = \frac{D}{R \sin(\ln(1+z)/\sqrt{2})}. \quad (5)$$

The angular size decreases with z until $z = 8.22$ where it has a broad minimum and then it increases with z till it becomes infinity at the antipole where $r = \pi R$ and $z = 84.02$.

3. The observations

The measurement of angular size of an object over a large range of distances has many problems. The first is that for an object to act as a standard measuring rod it must correctly identified at different distances. To a certain extent this can be avoided by observing a large number of objects that are drawn from a known statistical distribution. In the static model there is no evolution so that it is required that this distribution is the same (and hence the absolute size measurements are the same) at all distances. The second problem is the difficulty of measuring the size of objects such as galaxies with diffuse edges. To a large extent this is overcome in the observations of Djorgovski and Spinrad (1981) who used Petrosian (Petrosian 1976) radii of bright elliptic galaxies. They give the angular size and redshift for 25 galaxies. Taking a different approach Kapahi (1987) measured the separation of the lobes of radio galaxies. More recently this type of measurement been repeated by Nilsson *et al.*, (1993) who restricted their analysis to Faranoff-Riley type II double radio sources which have well defined edges so that the angular size is easily determined. Kellermann (1993) argued that compact radio sources observed with VLBI should be free of many of the selection biases and has provided results for a homogeneous

group of bright compact radio sources. These compact objects have a nominal size of 41 pc which is very small compared to a size of about 50 kpc for the optical galaxies and 400 kpc for the double radio sources. However his results do not agree with the other observations in that they differ markedly from the usual approximately Euclidean dependence on redshift. A problem with these sources is that their size is not unambiguously defined. Kellermann (1993) argues that they are not subject to systematic evolutionary effects and this explains the good agreement of their angular size with the standard model with $q_0 = 1/2$. Since Kellermann’s (1993) results differ greatly from the other data and clearly apply to quite different structures they are not considered here.

4. Analysis

The 540 double radio sources (all Faranoff-Riley type II) listed by Nilsson *et al.*, (1993) have been analysed using the static cosmological model. They assembled a new large sample from the literature using published radio maps to measure the angular size and to determine the morphology or where maps were not available they used the original data. The values for the linear size in the static cosmology were computed by multiplying the linear sizes for the standard model ($H_0 = \text{km.s}^{-1}.\text{Mpc}^{-1}$, $q_0 = 0$) given by Nilsson *et al.*, (1993,table(1)) by the distance ratio defined by

$$f_d = \frac{\sqrt{2} \sin(\ln(1+z)/\sqrt{2})}{\frac{1}{2} \left[1 - \frac{1}{(1+z)^2} \right]}.$$

Similarly the values for the radio luminosity were computed using

$$L_{static} = f_d^2 (1+z)^{-3} L_{standard}$$

where the factor in $1+z$ removes the redshift corrections that are not applicable in the static model. Note that the *tired light* model discussed by Nilsson *et al.*, (1993) and other authors differs from the static cosmology in that it has a Euclidean geometry which gives a distance proportional to $\ln(1+z)$ instead of proportional to $\sqrt{2} \sin(\ln(1+z)/\sqrt{2})$.

Nilsson *et al.*, (1993) found that for the standard cosmological model there is a strong dependence of luminosity as a function of redshift. Whereas the static model shows a much smaller dependence of luminosity on redshift. The average values of $\text{Log}(L)$ for all objects are shown in figure (1) for both the standard model and the static model. Using the geometric mean of the luminosities (ie., the average absolute magnitude) has the desirable

property that it is less sensitive to outliers than the arithmetic mean. The initial rise seen in figure (1) for both cosmological models is due to selection effects, in particular it is due to the selection bias of the nearby sources being weaker. In this heterogeneous sample of radio sources it is likely that most of them were discovered in flux density limited surveys. In addition there is usually a strong correlation in radio surveys between the flux density limit and the area of the survey in the sense that deep surveys with small flux density limits are usually confined to a small area of the sky whereas surveys that cover large areas invariably have larger flux density limits. The selection criteria is further compounded by the different frequencies that were used in the surveys. The major effect of the flux density limits and the area selection is the inclusion of an excessive number of nearby weak sources which strongly bias the absolute magnitudes to weaker values. The computed values for the average absolute magnitude and linear size (geometric average) as a function of redshift are shown in table (1). Because the data is assembled from the literature without any control of selection bias it is not clear whether the very slow increase in average luminosity calculated on the static model for $z > 0.5$ is a defect in the model or is due to selection effects. If the flux density selection limit is important it would be evident as a deficiency of weak sources at large distances which would show up as an increase in the average luminosity as z increases. The fact that the increase is so small for the static model suggests that flux density selection limit is not very important and that the static model is consistent with the data.

In order to see if there is a significant difference in the luminosities of galaxies versus the luminosities of quasars averages were taken for all objects with $z \geq 0.5$. For 69 galaxies the average of $\text{Log}(L)$ was 44.60 ± 0.05 and for 219 quasars it was 44.40 ± 0.03 . Although the small difference is statistically significant it is more interesting that the absolute magnitudes of radio loud quasars and galaxies are so similar.

The (projected) geometric average linear size (for $H_0 = 50 \text{ km.s}^{-1}.\text{Mpc}^{-1}$) of the radio sources is shown in figure (2) and in table (1) as a function of redshift. The small values at low redshifts reflect the same selection effects that are apparent in the average luminosities and are of little importance here. Except for the last point the static cosmology projected linear sizes are essentially constant for $z > 0.02$ which provides strong support for its validity. From table (1) the average linear size for $z > 2$ is 256 ± 32 kpc to be compared with the overall average of 315 ± 15 kpc for $0.02 < z < 2.0$. Although this is hardly statistically significant the discrepancy gains credence from considering the galaxies and quasars separately. For $z > 2.0$ there are 4 galaxies with an average linear size of 251 ± 89 kpc and 23 quasars with an average size of 257 ± 35 kpc. The agreement of these values suggests the the decrease in linear size is real. However without knowing the selection criteria for these sources it is too early to consider whether the decrease in size for the

highest redshift sources is to a defect in the static cosmology or unknown selection effects. This discrepancy in the highest redshift range is the strongest evidence against the static model. But because it is the extreme point and the numbers are small this discrepancy cannot be taken too seriously.

Kapahi (1987) concluded that based on angular size data there was no difference between galaxies and quasars. This is confirmed for the static model using the results of Nilsson *et al.*, (1993) and is consistent with them having similar absolute magnitudes. Using the static cosmological model and taking a redshift range of $0.02 \leq z \leq 2.0$ the geometric average linear size for 506 sources was 315 ± 15 kpc. In the same range the average size for galaxies was 315 ± 22 kpc and for quasars it was 315 ± 18 kpc. Closer examination of the results as a function of redshift show that there is no significant difference between the linear size of galaxies and that of quasars.

Since Nilsson *et al.*, (1993) found a power-size anticorrelation the correlation between the logarithm of the size and the luminosity was determined. For all 540 sources the correlation coefficient was 0.12 which is significant at the 0.5% level. However if the sources with sizes less than 30 kpc or greater than 2000 kpc are excluded the correlation coefficient for 513 sources drops to the insignificant value of 0.04 suggesting that the original correlation was an artifact of some extreme values. In any case the power-size anticorrelation is not found. The conclusion is that the occurrence of a significant power-size correlation coefficient is clearly dependent on the cosmological model that is used to determine the luminosities and linear sizes.

5. Comparison of all data

The angular size verses redshift data for all the radio and optical objects from Djorgovski and Spinrad (1981), Kapahi (1987) and Nilsson *et al.*, (1993) are shown in figure (3). They have all been normalized to have $\theta = 200''$ at $z = 0.07$ so that at small values of redshift they should agree. The four curves are for Euclidean, static (equation 5) (static) and from the standard theory with no evolution for $q_0 = 0$ and $q_0 = 1$. Considering the selection effects and measurement difficulties all the observations are in reasonable agreement. As already mentioned the observations differ markedly from the standard model (without evolution) at large redshifts. Nilsson *et al.*, (1993) provide a comprehensive discussion of the various evolutionary models that have been suggested and other alternatives such as a dependence of linear size on luminosity. In fact Nilsson *et al* (1993) argue that their data can be understood without cosmological evolution because of

a power-size anticorrelation. Although some of the models have some physical plausibility there is no general agreement on a suitable explanation that would bring the standard model into agreement with the observations. Whereas although the agreement with the static cosmological model is very much better it is not perfect. There appears to be a small discrepancy (discussed earlier) with the Nilsson *et al.*, (1993) results for the maximum redshift range of $z > 2$. Note that some of the radio sources used by Kapahi (1987) have had their redshifts determined from optical magnitudes and in general his data have been superseded by that from Nilsson *et al.*, (1993).

6. Conclusion

It has been shown that the static cosmological model provides a good fit to angular size data for large radio radio galaxies, quasars and optical galaxies. There is also support from the luminosity data shown in figure (1) for radio loud quasars and galaxies, in that if the model is correct and selection effects are small the luminosity should be independent of redshift. What is observed is apart from nearby sources where there are obvious and expected selection effects the luminosity is almost constant. There is a small anomaly in the angular size comparison at large redshifts but since this could be due to section effects there is no unequivocal evidence that would cause the static cosmology to be rejected.

7. Acknowledgements

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Table 1: $\text{Log}(L/\text{erg sec}^{-1})$ and linear size as a function of redshift.

z range	\bar{z}	number	$\text{Log}(L)$	Linear size (D)
$0.002 \leq z < 0.005$	0.0030	2	39.50 ± 0.60	3 ± 5
$0.005 \leq z < 0.01$	0.0060	3	41.20 ± 0.53	48 ± 67
$0.01 \leq z < 0.02$	0.0165	7	40.95 ± 0.39	180 ± 114
$0.02 \leq z < 0.05$	0.036	23	42.07 ± 0.12	307 ± 65
$0.05 \leq z < 0.14$	0.072	54	42.36 ± 0.10	279 ± 47
$0.1 \leq z < 0.2$	0.147	45	43.01 ± 0.09	330 ± 53
$0.2 \leq z < 0.3$	0.244	44	43.54 ± 0.06	312 ± 49
$0.3 \leq z < 0.5$	0.398	74	43.80 ± 0.05	323 ± 44
$0.5 \leq z < 0.8$	0.65	80	44.17 ± 0.05	313 ± 37
$0.8 \leq z < 1.0$	0.88	48	44.40 ± 0.06	325 ± 42
$1.0 \leq z < 1.5$	1.20	79	44.52 ± 0.05	323 ± 34
$1.5 \leq z < 2.0$	1.68	54	44.67 ± 0.06	318 ± 41
$2.0 \leq z < 5.0$	2.57	27	44.71 ± 0.08	256 ± 32

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Fig. 1.— Average $\text{Log}(L/\text{erg sec}^{-1})$ versus z for double radio sources for standard ($q_0 = 0$) and static cosmologies

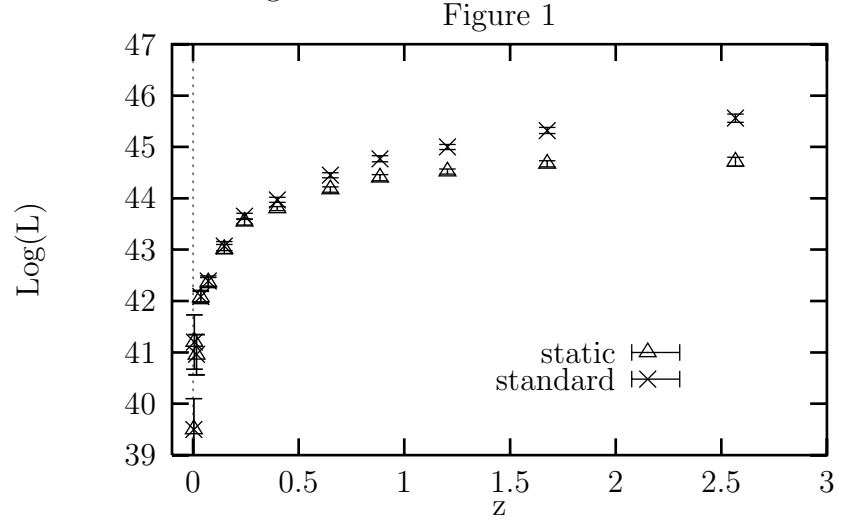


Fig. 2.— Average linear size (kpc) for double radio sources vs. redshift for standard ($q_0 = 0$) and static cosmologies

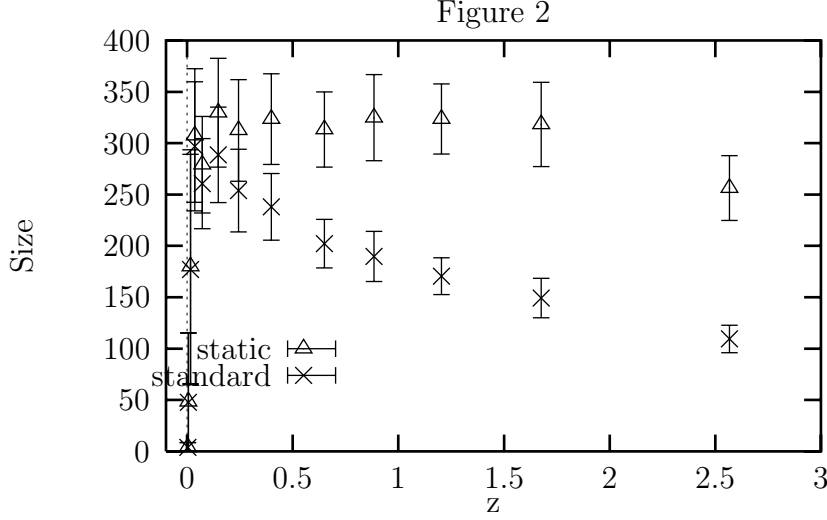


Fig. 3.— Angular size versus redshift: \diamond - Kapahi (1987), $*$ - Djorgovski and Spinrad (1981), \circ - Nilsson *et al.*, (1993) galaxies and \bullet - quasars. Curves are for for standard ($q_0 = 0, q_0 = 1$), static and Euclidean cosmologies.

